INSTITUTE OF MATHEMATICS AND APPLICATIONS, BHUBANESWAR

Entrance Test for Admission into M.A./M.Sc. Courses
(Mathematics with Data Science / Computational Finance)

2022-2023

Maximum Marks: 100 Time: 2 Hours.

INSTRUCTIONS

- Ensure that this Test Booklet contains 06 printed pages with 50 multiple choice questions.
- Candidates are required to check that the Test Booklet does not have any discrepancy (ies) like un-printed or torn or missing pages, missing questions etc., if so, get it replaced by a complete test booklet before attempting to answer. No extra time will be given, if replaced afterwards.
- Each of the questions/incomplete statements is followed by four options/choices marked as (A), (B), (C), (D) under each question/statement, of which only one of them is correct/most appropriate.
- For each question, mark the correct/most appropriate option/choice by putting a cross (×) mark in the appropriate box of the answer sheet provided to you. In case, a candidate feels that there is multiple correct options/choices, the candidate has to mark the option/choice which he/she feels is the most appropriate/best. In any case, only one option/choice has to be marked for each question. More than one option/choice marked in the answer sheet against a question number will be deemed as incorrect.
- If you mark your option/choice at any place other than the box provided, it will not be evaluated.
- Each correct answer carries 2 marks.
- Use of any written/printed material, calculator, docu-pen, any communication devices like cell phones/i-pods etc. inside the examination hall is not allowed. Candidates found with such items will be reported and his/her candidature will be summarily cancelled.
- Blank Sheet(s) for doing rough work/calculations is/are appended at the end of the Test Booklet.

Warning: Any malpractice or any attempt to commit any kind of malpractice in the examination hall will disqualify the candidate

QUESTIONS

1. What is the rank of the matrix $\begin{pmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{pmatrix}$?

(A)0 (B) 1 (C) 2 (D) 3

2. The eigenvectors corresponding to distinct eigenvalues of a matrix are

(A) Orthogonal (B) Linearly dependent (C) Linearly independent (D) None of these

3. Which of the following set of vectors are linearly independent?

(A) { (2,3,1),(2,1,3),(1,1,1)} (B) { (0,-1,3),(3,4,3),(1,1,2)}

(C) { (2,3,1),(1,0,1),(1,1,2)} (D) All of these.

4. The order of a proper subgroup of a group G of order 19 is

(A) 1 (B) 19 (C) 1 or 3 (D)None of these

5. Which of the following is not correct?

(A) Every cyclic group is an abelian group.

(B)Every group of odd order is cyclic.

(C)The order of a cyclic group and the order of its generating elements is equal.

(D)Every subgroup of a cyclic group is cyclic.

6. The number of generators of a cyclic group of order 6 is

(A) 6 (B) 2 (C)3 (D)1

7. If p is a prime and a is any number prime to p , then p divides

(A) $a^p - 1$ (B) a^{p-1} (C) $a^{p-1} - 1$ (D) $a^p + 1$

8. Newton-Raphson iteration formula for finding $\sqrt[3]{c}$, where c > 0 is

(A) $x_{n+1} = \frac{2x_n^3 + \sqrt[3]{c}}{2x_n^2}$ (B) $x_{n+1} = \frac{3x_n^3 - \sqrt[3]{c}}{3x_n^2}$ (C) $x_{n+1} = \frac{2x_n^3 + c}{3x_n^2}$ (D) $x_{n+1} = \frac{2x_n^3 - c}{3x_n^2}$

9. Which one of the following results is incorrect?

(A) $\Delta x^n = nx^{n-1}$ (B)) $\Delta x^{n-1} = nx^{n-1}$ (C)) $\Delta^n e^x = e^x$ (D) $\Delta cosx = -sinx.$

10. P V (P→Q) V { ~(P V Q)} is a

(A) Tautology (B) Contradiction (C) Contingency (D) None of these.

11. Let f(x) denote the objective function of an LPP, then

(A) Max f(x)= -min{-f(x)}   (B) Max f(x)= -min{f(-x)}

(C) Max f(x)= -min{f(x)}   (D)) Max f(x)= min{-f(-x)}

12. Given, $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ the maximum number of possible basic solutions is

(A) 3   (B) 4   (C) 2   (D) 6

13. Choose Maclaurin's series expansion of cos x from the following alternatives.

(A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\cdots\cdots \quad \forall x \in R.$

(B) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\cdots\cdots \quad \forall x \in R.$

(C) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots\cdots \quad \forall x \in R.$

(D) None of the above.

14. If a real number $c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfies the conclusion of Rolle's theorem for the function f(x)=cosx in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ then what is the value of c?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) π (D) 0

15. If some c∈ (2 , 4) satisfies the conclusion of Lagrange's mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ in [2, 4] then what is the value of c?

(A) 12   (B) 6   (C) $\sqrt{2}$ (D) $\sqrt{6}$

16. The real part of the complex number $(1 + i)^n$ is :

(A) $2^{\frac{n}{2}} \cos\frac{n\pi}{4}$ (B) $2^n \cos\frac{n\pi}{2}$ (C) $2^{-\frac{n}{2}} \cos n\pi$ (D)) $2^{-n} \cos\frac{n\pi}{2}$

17. If α, β are root of the equation $x^2 - 2x + 4 = 0$, *then* $\alpha^n + \beta^n$ is equal to

(A) $2^{n+1}\cos(\frac{n\pi}{3})$ (B) $2^n\cos(\frac{n\pi}{3})$ (C) $2^{n+1}\sin(\frac{n\pi}{3})$ (D) $2^n\sin(\frac{n\pi}{3})$

18. If H_1 and H_2 are two subgroups of G , then which of the following is also a subgroup of G?

(A) $H_1 \cap H_2$ (B) $H_1 \cup H_2$ (C) $H_1 H_2$ (D)None of these.

19. If G is a group such that a^2=e , for all a∈ G, then G is

(A) an abelian group   (B) a non-abelian group   (C) a ring   (D) a field

20. In the group ({ 0,1,2,3,4},$+_5$), the order of 2 is

(A) one   (B) two   (C) three   (D)four.

21. If f is a homomorphism from (Z,+) to (R^+ , *) , and f(2) = 1/3 , then the value of f(6) is

(A) 1/27   (B) 27   (C) 81   (D) 1/81.

22. The homogeneous differential equation M (x , y) dx + N (x , y)dy =0 can be reduced to a differential equation in which the variable are separated ,by the substitution

(A) yx = v (B) y = vx (C) x + y = v (D) x – y = v.

23. $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$ has the solution

(A) $y=c_1e^{-2x}+c_2e^{x}$ (B) ce^{-2x} (C) $y=c_1e^{-2x}+c_2e^{-x}+c_3$ (D) None of these.

24. The integrating factor for the differential equation $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$ is

(A) 1/(x+1) (B) x+1 (C) $1/(x^2+1)$ (D) x^2+1

25. If y_1 and y_2 are two solutions of the initial value problem $y'' + p(x)y' + q(x)y = 0$, $y(x_0)=y_0$, $y'(x_0)=y_0$ and the wronskian $W(y_1, y_2)=0$ then y_1 and y_2 are

(A) Linearly dependent (B) Linearly independent (C) Proportional (D) None of these.

26. The differential equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=50e^{2x}$ has particular integral

(A) $(2e^{2x})/3$ (B) $2e^{2x}$ (C) e^{2x} (D) None of these.

27. Three houses are available in a locality. 3 persons apply for the house . Each applies for one house without consulting others . The probability that all the three apply for the same house is

(A) 2/9 (B) 1/9 (C) 8/9 (D) 7/9.

28. The probability that a teacher will give an unannounced test during any class is 1/5 . If a student is absent twice, then the probability that he misses at least one test is

(A) 2/5 (B) 4/5 (C) 7/25 (D) 9/25.

29. A test has 50 questions with four choices for each answer, one and only one being correct. Answers to all questions are chosen at random. If 2 marks are awarded for each correct answer and -1 is awarded for an incorrect answer the probability that a candidate with secure 40 marks is

(A) $\frac{3^{20}}{4^{50}}$ (B) $50_{C_{30}}\times\frac{3^{20}}{4^{50}}$ (C) 4/5 (D) None of these.

30. A rifleman is firing at a distant target and has only 10% chance of hitting it. The number of rounds, he must fire in order to have 50% chance of hitting it at least once is

(A) 11 (B) 9 (C) 7 (D) 5.

31. The random variable X takes the value 0,1,2 and 3 with probability 1/125 , 123/125 and 64/125 Respectively. Then the value of E(x) is

(A) 12/4 (B) 12/7 (C) 12/5 (D) None of these.

32. Given the probability density function $f(x)=\begin{cases}e^{-x}, x\geq 0\\ 0, x<0\end{cases}$ then the value of Cumulative Distribution function at x=2 is

(A) $1+e^{-2}$ (B) e^{-2} (C) e^{-1} (D) $1-e^{-2}$.

33. If $F(x) = \int_1^x \frac{dt}{(t^2+1)^2}$, then F'(1)=........

(A) 0 (B) -1/2 (C) 1/4 (D) None of these.

34. x^x is a minimum for

(A) x=e (B) x = 1/e (C) x = 1 (D) x = 1/10.

35. The area above the x-axis included between the parabola y^2 =ax and the circle $x^2 + y^2$ =2ax is

(A) $\int_{-a}^{a}(ax - x^2)dx$ (B) $\int_0^a(\sqrt{2ax} - \sqrt{ax})dx$

(C) $\int_{-a}^{a}(\sqrt{2ax} - \sqrt{ax})dx$ (D) $\int_0^{2a}(\sqrt{ax} - \sqrt{2ax})dx$.

36. $\int_0^1 \frac{dx}{x^{\frac{2}{3}}}$

(A) diverges to $+\infty$ (B) diverges to $-\infty$ (C) is equal to 0 (D) None of these.

37. If we change the order of integration, $\int_0^1(\int_0^{1-y} f(x,y)dx)dy$ becomes

(A) $\int_0^{1-y}(\int_0^1 f(x,y)dx)dy$ (B) $\int_0^{1-y}(\int_0^1 f(x,y)dy)dx$

(C) $\int_0^1(\int_0^{1-x} f(x,y)dx)dy$ (D) $\int_0^1(\int_0^{1-x} f(x,y)dy)dx$.

38. If $f(x,y) = \begin{cases} \frac{x-y}{x+y}, & for\ x \neq -y \\ 1, & for\ x = -y \end{cases}$ then as (x, y) → (0 , 0), f (x , y) approaches

(A) 1 (B) -1 (C) 0 (D) no limit.

39. $\lim_{x\to 0}(1 + x)^{\frac{1}{x}} = \cdots$

(A) 0 (B) 1 (C) e (D) None of these.

40. Let x ∊ R . Then $|2x + 1|$ is differentiable

(A) on R (B) on R-{0} (C) on R-{1/2} (D) on R- {-1/2}.

41. The function f is continuous on [0 , 1] such that f(0)= -2 , f(1/2)= ½ , and f (1)= -1, we can then conclude that

(A) f is non-zero on [0 , 1] (B) f vanishes at least twice on [0 , 1]

(C)) f vanishes exactly twice on [0 , 1] (D)) f vanishes exactly once on [0 , 1].

42. A sequence { x_n } is monotonic and bounded ,then

(A) all subsequences of { x_n } converge to the same limit.

(B) there exist a subsequence that diverges

(C) there exist at least two subsequences { x_n } which converge to distinct points

(D)None of these.

43. The least upper bound of the set $\{\frac{n+1}{n}, n \in N\}$ is

(A) 2 (B) 1 (C) 0 (D) non existent

44. If $\lim_{n\to\infty} a_n = 1$ then the series $\sum_{n=1}^{\infty} a_n$

(A) diverges to $+\infty$ (B) diverges to $-\infty$ (C) oscillates (D) converges.

45. The series $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha-1}}$ is convergent if

(A) $\alpha > 1$ (B)) $\alpha \leq 1$ (C)) $\alpha > 2$ (D)) $\alpha \leq 2$.

46. The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\cdots$

A) converges to e (B) converges to ln(1/2) (C) diverges (D) converges to ln2.

47. $\sum_{n=0}^{\infty} \frac{n+1}{2^n} =$

(A) 4/9 (B) 1 /4 (C) 4 (D) 2.

48. The set of integers as a subset of the set of real numbers is

(A) a closed set (B) an open set (C) neither open nor closed (D) open and closed .

49. The set S= { 1 , ½ , 1/3 , ...} $\subset R$ is

(A) closed (B) open (C) neither open nor closed (D) Both open and closed.

50. Which of the following statements is false?

(A) a function which is differentiable at a point is continuous at that point.

(B) a function continuous on an interval [a , b] is integrable on [a , b]

(C) a continuous function having countable number of discontinuities on [a , b] is integrable.

(D) a function continuous on [a , b] is differentiable on [a , b].