

**INSTITUTE OF MATHEMATICS & APPLICATIONS BHUBANESWAR**  
**ENTRANCE TEST FOR ADMISSION INTO M.A./M.Sc. COURSES**  
**(MATHEMATICS WITH DATA SCIENCE,**  
**AND COMPUTATIONAL FINANCE)**  
2021 – 2022

Maximum Marks: 100

Time: 2 Hours

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Please read carefully, the instructions that follow.

- This question booklet contains 10 numbered and printed pages with 50 multiple choice questions.
- Check that the booklet does not have any unprinted or torn or missing pages or questions etc. If so, get it replaced by a complete test booklet before attempting to answer. No extra time will be given, if replaced afterwards.
- Each of the questions/incomplete statements is followed by four options or choices marked as (A), (B), (C), (D) under each question of which only one of them is correct/most appropriate.
- Answer each question by writing correct choice A/B/C/D, in the space provided for it in the answer booklet, which is being supplied separately. This question paper is meant to be retained by you and so do not answer questions on it.
- More than one choice marked against a question will be deemed as incorrect.
- Each correct answer carries 2 marks.
- Use of calculator, log table, mobile phone or any electronic gadget in the examination hall is not allowed.

**Warning:** *Any malpractice or any attempt to commit any kind of malpractice in the examination hall will disqualify the candidate.*

## NOTATIONS

|                             |   |
|-----------------------------|---|
| $\mathbb{N}$                | set of all natural numbers $\{1, 2, 3, \dots\}$ .   |
| $\mathbb{Z}$                | set of all integers $\{0, \pm 1, \pm 2, \dots\}$ .  |
| $\mathbb{Q}$                | set of all rational numbers.  |
| $\mathbb{R}$                | set of all real numbers.  |
| $\mathbb{C}$                | set of all complex numbers.   |
| $\mathbb{R}^n$              | $n$ -dimensional Euclidean space $\{(x_1, x_2, \dots, x_n) : x_k \in \mathbb{R}, 1 \leq k \leq n\}$ .   |
| $\mathbb{C}^n$              | $n$ -dimensional Euclidean space $\{(x_1, x_2, \dots, x_n) : x_k \in \mathbb{C}, 1 \leq k \leq n\}$ .   |
| $\mathbb{Z}_n$              | The set $\{0, 1, \dots, n - 1\}$ for $n \geq 1$ is a group under addition modulo $n$ .  |
| $M_n(\mathbb{R})$           | real vector space of all matrices of order $n \times n$ with entries in $\mathbb{R}$ .  |
| $\hat{i}, \hat{j}, \hat{k}$ | unit vectors having directions of the positive $X, Y$ and $Z$ axes of a three dimensional rectangular coordinate system.  |
| $\binom{n}{r}$              | Denote the standard binomial coefficient giving the number of ways of choosing $r$ objects from a collection of $n$ objects, where $n \geq 1$ and $0 \leq r \leq n$ are integers. |

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**MULTIPLE CHOICE QUESTIONS**

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1. Consider the following system of equations:

$$x_1 + 3x_2 + 3x_3 = 5$$

$$3x_1 + 3x_2 + 2x_3 = 1$$

$$4x_1 - 2x_3 = b.$$

For which value of  $b$  is the system consistent?

- (A) -5                      (B) 3                      (C) -8                      (D) 4
2. Consider the following sets of vectors in  $\mathbb{R}^3$ .
- I:  $\{x \mid x = (x_1, x_2, x_3) \in \mathbb{R}^3, x_1 \geq 0\}$ ;
- II:  $\{x \mid x = (x_1, x_2, x_3) \in \mathbb{R}^3, 3x_1 - x_2 = x_3\}$ ;
- III:  $\{x \mid x = (x_1, x_2, x_3) \in \mathbb{R}^3, x_1 = x_2^2\}$ ;
- IV:  $\{x \mid x = (x_1, x_2, x_3) \in \mathbb{R}^3, x_1x_2 = 0\}$ .
- Choose the correct option which is (are) subspace(s) of  $\mathbb{R}^3$ .
- (A) I and II only.      (B) II only.                      (C) II and IV only.      (D) II, III and IV.
3. If  $\{v_1, v_2, v_3\}$  is a linearly independent set of vectors in a vector space over  $\mathbb{R}$ , then which one of the following sets is also linearly independent?
- (A)  $\{v_1 + v_2 - v_3, 2v_1 + v_2 + 3v_3, 5v_1 + 4v_2\}$ .
- (B)  $\{v_1 - v_2, v_2 - v_3, v_3 - v_1\}$ .
- (C)  $\{v_1 + v_2 - v_3, v_2 + v_3 - v_1, v_3 + v_1 - v_2, v_1 + v_2 + v_3\}$ .
- (D)  $\{v_1 + v_2, v_2 + 2v_3, v_3 + 3v_1\}$ .
4. Let  $A$  be an  $m \times n$  matrix with entries in field  $F$  and let  $T$  be a linear transformation from  $F^{n \times 1}$  into  $F^{m \times 1}$  defined by  $T(X) = AX$ . Choose the correction option from the following:
- (A) If  $m < n$ , then  $T$  is one-one and onto.
- (B) If  $m > n$ , then  $T$  is one-one and onto.
- (C) If  $m < n$ , then  $T$  is onto but not one-one.
- (D) If  $m > n$ , then  $T$  is onto but not one-one.

5. If  $A$  and  $B$  are real symmetric matrices of size  $n \times n$ , then which one of the following is correct?

- (A)  $AA^T = I$ .      (B)  $A = A^{-1}$ .      (C)  $AB = BA$ .      (D)  $(AB)^T = BA$ .

6. Let  $A = \begin{pmatrix} 1 & 4 & 8 & 7 \\ 0 & 12 & 24 & 21 \\ 0 & 0 & 3 & 0 \\ 4 & 0 & 0 & 0 \end{pmatrix}$ . Choose the correct option from the following:

- (A) Rank  $(A) = 4$ .    (B) Rank  $(A) = 3$ .    (C) Rank  $(A) = 2$ .    (D) Rank  $(A) = 1$ .

7. Let  $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ . Then the product of eigenvalues of  $A$  is

- (A) 1.                      (B) 2.                      (C) 4.                      (D) 8.

8. Which of the following statement(s) is/are **TRUE** ?

- I. The set  $\{1, 2, 3\}$  under multiplication modulo 4 is a group.  
 II. The set  $\{1, 2, 3, 4\}$  under multiplication modulo 5 is a group.

- (A) only I.                      (B) only II.  
 (C) I and II.                      (D) None.

9. Let  $Z_n$  be a cyclic group of order  $n$ . What is the total number of elements of order 8 in  $Z_{8000}$  ?

- (A) 32.                      (B) 16.                      (C) 8.                      (D) 4.

10. Let  $H = \{0, \pm 5, \pm 10, \pm 15, \dots\}$ . What is the total number of left cosets of  $H$  in the group  $\mathbb{Z}$ ?

- (A) 15.                      (B) 10.                      (C) 5.                      (D) 1.

11. Let  $\sigma$  be an element of the permutation group  $S_7$ . What is the maximum possible order of  $\sigma$ ?

- (A) 7.                      (B) 10.                      (C) 12.                      (D) 15.

12. Which of the following statement(s) is/are **TRUE** ?

- I. Up to isomorphism, there are two abelian groups of order 6.  
 II. Up to isomorphism, there are two non-abelian groups of order 6.  
 III. Up to isomorphism, there is one abelian group of order 6 and one non-abelian group

of order 6.

- (A) I. (B) II.  
(C) III. (D) None.

13. How many elements are there in the ring  $\mathbb{Z}[i]/\langle 3+i \rangle$ ?

- (A) 3. (B) 6. (C) 9. (D) 10.

14. Which of the following is an equivalence relation?

- (A) The inclusion relation on the set of sets.  
(B) The relation "has same number of vertices" on the set of all polygons.  
(C) The relation "is the square of" on the set of positive integers.  
(D) The relation "is similar to" on the set of all triangles.

15. Choose the **incorrect** set theoretic identity from the following.

- (A)  $A \setminus (B \cup C) = (A \setminus B) \setminus C$ .  
(B)  $(A \setminus B) \cap C = A \setminus (B \cap C)$ .  
(C)  $(A \setminus B) \cap (C \setminus D) \subset (A \cap C) \setminus (B \cap D)$ .  
(D)  $A \setminus (B \cup C) = (A \setminus B) \setminus C$ .

16. Choose the **correct** relation from the following.

- (A)  $2^n > n!$  for every  $n \in \mathbb{N}$ . (B)  $n^2 > 2^n$  for every  $n \in \mathbb{N}$ .  
(C)  $n > 2^n$  for every  $n \in \mathbb{N}$ . (D)  $n! > n^n$  for large  $n \in \mathbb{N}$ .

17. In a party there were  $n$  number of guests. If each guest shook hands only once with rest others, how many hand shakes took place?

- (A)  $n^2$ . (B)  $\frac{n^2}{2}$ . (C)  $n(n-1)$ . (D)  $\frac{n(n-1)}{2}$ .

18. Let  $[x]$  denote the greatest integer  $\leq x$ . Choose the **correct** relation from the following.

- (A)  $[x+n] = [x] + n$  for every  $n \in \mathbb{N}$ .  
(B)  $[x+y] = [x] + [y]$  for every  $x, y \in \mathbb{R}$ .  
(C)  $[2x] = 2[x]$  for every  $x \in \mathbb{R}$ .  
(D)  $[-x] = -[x]$  for every  $x \in \mathbb{R}$ .

19. Choose the **correct** option to complete the given statement. A Cauchy sequence  $a_n$  in a discrete metric space.....
- (A) has infinitely many distinct terms.
  - (B) has only finitely many distinct terms.
  - (C) has all the terms equal to each other.
  - (D) has all  $a_n$ 's distinct.
20. Let  $(X, d)$  be a metric space. For  $a \in X$  and  $r > 0$  define a ball  $B(a, r) = \{x \in X : d(x, a) < r\}$ . Choose the **incorrect** option from the following.
- (A) If  $x, y \in B(a, r)$  then the straight line segment  $tx + (1 - t)y$ ,  $(0 \leq t \leq 1)$  is contained in  $B(a, r)$ .
  - (B) A ball in a metric space need not always contain infinitely many points.
  - (C) If  $B(a_1, r) = B(a_2, s)$  then  $a_1 = a_2$  and  $r = s$ .
  - (D) If  $x, y \in X$  then there exists  $r > 0$  such that  $B(x, r) \cap B(y, s) = \emptyset$ .
21. Choose the **correct** option from the following. Let  $\mathbb{R}$  have the usual metric. For a subset  $A$  of  $\mathbb{R}$  let  $A^\circ$  denote the set of interior points of  $A$ .
- (A) Then  $\mathbb{Q}^\circ = \mathbb{R}$ .
  - (B) Then  $\mathbb{Z}^\circ = \emptyset$ .
  - (C) Then  $(a, b)^\circ = [a, b]$ .
  - (D) Then  $[a, b]^\circ = [a, b]$ .
22. Complete the following statement by choosing the **correct** option. In a metric space arbitrary union of.....
- (A) connected sets is a connected set.
  - (B) compact sets is a compact set
  - (C) open sets is an open set.
  - (D) closed sets is a closed set.
23. Complete the following statement by choosing the **correct** option. If  $z$  is a complex number and  $|z| < 1$  then  $\Re\left(\frac{1+z}{1-z}\right)$ .....
- (A)  $> 0$ .
  - (B)  $< 0$ .
  - (C)  $= 0$ .
  - (D)  $= \infty$ .
24. Complete the following statement by choosing the **correct** option. If  $a$  and  $b$  are complex numbers then equality holds in triangle inequality if and only if.....
- (A)  $a$  and  $b$  lie in a straight line.
  - (B)  $a = rb$  for some real number  $r$ .

- (C)  $\arg a = \arg b$   
 (D)  $a$  and  $b$  are both positive real numbers.
25. Choose the correct option from the following.  
 (A)  $\sin i$  is a real number.  
 (B)  $i \sin i$  is a negative real number.  
 (C)  $i \sin i$  is a positive real number.  
 (D)  $i \sin i$  is an imaginary number.
26. Which one of the following is equal to  $\limsup_{n \rightarrow \infty} \sin\left(\frac{n\pi}{3}\right)$ ?  
 (A)  $\sqrt{3}/2$ .                      (B) 0.                      (C) 1.                      (D)  $-\sqrt{3}/2$ .
27. What are the integral curves of the equation
- $$\frac{dr}{d\phi} = r(r-1)(r-2)$$
- where  $r$  and  $\phi$  are polar coordinates?
- (A) Three straight lines.  
 (B) Two concentric circles and a straight lines.  
 (C) Two straight lines and a circle.  
 (D) Two concentric circles and a spiral.
28. Which of the following equation is homogeneous in  $x, y$ ?  
 (A)  $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$ .  
 (B)  $(x^2 - 2y^2)dx + xy dy = 0$ .  
 (C)  $(xe^{xy} + 2y)dy + ye^{xy} dx = 0$ .  
 (D)  $(3x^2y + 8xy^2)dx + (x^3 + 8x^2y + 12y^2)dy = 0$ .
29. Which of the following is the complimentary function of  $(D^2 + 4)y = \tan 2x$ ?  
 (A)  $A \cos 2x + B \sin 2x$ .                      (B)  $A \cos 2x - B \sin 2x$ .  
 (C)  $A \cosh 2x + B \sinh 2x$ .                      (D)  $A \cosh 2x - B \sinh 2x$ .
30. If the roots of an auxilliary equation are  $(-5, -5)$  then which one of the following is the particular integral?  
 (A)  $(A + Bx)e^{-5x}$ .                      (B)  $(A - Bx)e^{-5x}$ .  
 (C)  $(A + Bx)e^{-10x}$ .                      (D)  $(A - Bx)e^{-10x}$ .

31. What is the value of the Wronskian for  $y_1 = \sin 2x$  and  $y_2 = \cos 2x$ ?  
 (A) 1. (B)  $-1$ . (C) 2. (D)  $-2$ .
32. Which of the following is the general solution of the equation  $\frac{d^2y}{dx^2} + 8y = 0$ ?  
 (A)  $y = Ae^{(2\sqrt{2}x)} + Be^{-(2\sqrt{2}x)}$ . (B)  $y = Ae^{(2\sqrt{2}x)} - Be^{-(2\sqrt{2}x)}$ .  
 (C)  $y = A \cos(2\sqrt{2}x) + B \sin(2\sqrt{2}x)$ . (D)  $y = A \cos(2x) + B \sin(2x)$ .
33. Which of the following is the general solution of the equation  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{-x}$ ?  
 (A)  $y = (C_1 + C_2x)e^x + \frac{1}{2}e^{2x}$ . (B)  $y = (C_1 + C_2x)e^x + \frac{1}{4}e^{-x}$ .  
 (C)  $y = C_1e^x + C_2e^{-x}$ . (D)  $y = C_1xe^x + C_2xe^{-x}$ .
34. If  $J_n(x)$  is the Bessel function of first kind of order  $n$  then which of the following is true?  
 (A)  $J_n(x)$  and  $J_{-n}(x)$  are linearly independent.  
 (B)  $J_n(x) = J_{-n}(x)$ .  
 (C)  $J_n(x) = (-1)^n J_{-n}(x)$ .  
 (D)  $J_n(x) = nJ_{-n}(x)$ .
35. In two samples, the first sample has 100 items with mean 15 and standard deviation 3. The combined sample has 250 items, with mean 15 and standard deviation  $\sqrt{13.44}$ . What is the standard deviation of the second sample?  
 (A) 4. (B) 3. (C) 2. (D) 1.
36. Complete the following statement by choosing the correct option. For any frequency distribution Kurtosis is .....  
 (A)  $> 1$ . (B)  $< 1$ . (C)  $= 1$ . (D)  $\infty$ .
37. An urn contains 6 red and 4 black balls. Two balls are drawn without replacement. What is the probability that the second ball is red if it is known that the first is red?  
 (A)  $\frac{6}{10}$ . (B)  $\frac{1}{5}$ . (C)  $\frac{5}{9}$ . (D)  $\frac{1}{3}$ .
38. A random variable  $X$  has p.d.f.

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$$

What is  $E(X)$ ?

- (A)  $\frac{b-a}{2}$ . (B)  $\frac{(a-b)^2}{2}$  (C)  $\frac{a+b}{2}$  (D)  $\frac{(a+b)^2}{2}$ .

39. Let  $X$  be a continuous random variable with p.d.f.  $f(x) = Ce^{-\frac{x}{\sigma}}$ ,  $x \geq 0$ ,  $\sigma > 0$ . What is its mean?  
 (A)  $\sigma^2$ . (B)  $-\sigma^2$  (C)  $\sigma$  (D)  $-\sigma$
40. If vectors are drawn from the centre of a regular polygon to its vertices, then what is the sum of these vectors?  
 (A)  $\vec{0}$ . (B) unit vector.  
 (C) any nonzero vector (D) undefined.
41. If  $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$  and  $\vec{b} = -\hat{i} + 2\hat{j} - 3\hat{k}$  then what is the value of  $\vec{a} \times \vec{b}$ ?  
 (A)  $\hat{i} + \hat{j} + \hat{k}$ . (B)  $-\hat{i} + \hat{j} + \hat{k}$  (C)  $\hat{i} - \hat{j} + \hat{k}$  (D)  $\hat{i} + \hat{j} - \hat{k}$
42. If  $\mathbf{u}(t)$  is a vector function of constant magnitude and  $\frac{d\mathbf{u}}{dt} \neq 0$  then which of the following is **true**?  
 (A)  $\frac{d\mathbf{u}}{dt}$  is perpendicular to  $\mathbf{u}(t)$ .  
 (B)  $\frac{d\mathbf{u}}{dt}$  is parallel to  $\mathbf{u}(t)$ .  
 (C) angle between  $\mathbf{u}(t)$  and  $\frac{d\mathbf{u}}{dt}$  is  $\frac{\pi}{4}$   
 (D) angle between  $\mathbf{u}(t)$  and  $\frac{d\mathbf{u}}{dt}$  is  $\frac{\pi}{6}$ .
43. What is the value of the divergence of the vector field  $\mathbf{u}(t) = y^2\hat{i} + 2xy\hat{j} - z^2\hat{k}$  at the point  $(1, 2, 1)$ ?  
 (A)  $-4$ . (B)  $0$  (C)  $4$  (D)  $5$ .
44. A function  $f(x)$  is defined as follows.

$$f(x) = \begin{cases} \sin x, & x \leq c \\ ax + b, & x > c. \end{cases}$$

where  $a$ ,  $b$ ,  $c$  are constants and  $c \neq 0$ . If  $b$  and  $c$  are given find the value of  $a$  for which  $f(x)$  is continuous at the point  $x = c$ .

- (A)  $\frac{\sin c - b}{c}$ . (B)  $\frac{\sin c - b}{c^2}$   
 (C) at any real number  $c$  (D)  $f$  cannot be continuous at  $c$  for any real number  $x$ .
45. Suppose that  $f'(a)$  exists. Consider the following.

I.  $f'(a) = \lim_{h \rightarrow a} \frac{f(h) - f(a)}{h - a}$ .

II.  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a) - f(a - h)}{h}$ .

$$\text{III. } f'(a) = \lim_{h \rightarrow 0} \frac{f(a+2h) - f(a)}{h}$$

$$\text{IV. } f'(a) = \lim_{h \rightarrow 0} \frac{f(a+2h) - f(a+h)}{2h}$$

Which of the above is/are true?

- (A) only I. (B) I and II only.  
 (C) I, II, and III only. (D) None.
46. Which one is the scheme for finding the square root of positive integer using the Newton-Raphson method
- (A)  $x_{n+1} = x_n + \frac{x_n^2 - N}{2x_n}$  (B)  $x_{n+1} = x_n - \frac{x_n^2 + N}{2x_n}$   
 (C)  $x_{n+1} = x_n + \frac{x_n^2 + N}{2x_n}$  (D)  $x_{n+1} = x_n - \frac{x_n^2 - N}{2x_n}$ .
47. What is the order of convergence in the secant method?
- (A) 1.235. (B) 3.141 (C) 1.618 (D) 2.
48. In which of the following interval the root of the equation  $x^3 - 2x - 5 = 0$  lies ?
- (A)  $(-1, 1)$ . (B)  $(2, 3)$  (C)  $(-1, 0)$  (D)  $(1, 2)$
49. For which of the following polynomials Simpson's 1/3 rule is exact?
- (A) cubic polynomial. (B) Quadratic polynomial.  
 (C) linear polynomial (D) none of these.
50. Find the value of the integral  $\int_0^1 \frac{dx}{1+x^2}$  using Trapezoidal rule with  $h = 0.5$ .
- (A) 0.071. (B) 0.565 (C) 0.775 (D) 1.78
-