INSTITUTE OF MATHEMATICS AND APPLICATIONS, BHUBANESWAR
ENTRANCE TEST FOR ADMISSION INTO B.Sc.(HONOURS) IN MATHEMATICS & COMPUTING, 2022-2023

Maximum Marks: 100 **Time: 2 hours**

INSTRUCTIONS TO CANDIDATES.

- Ensure that this Test Booklet contains 08 printed pages with 50 multiple choice questions.
- Candidates are required to check that the Test Booklet does not have any discrepancy(ies) like un-printed or torn or missing pages, missing questions etc., if so, get it replaced by a complete test booklet before attempting to answer. No extra time will be given, if replaced afterwards.
- Each of the questions/incomplete statements is followed by **four** options/choices marked as **(A)**, **(B)**, **(C)**, **(D)** under each question/statement, of which only one of them is correct/most appropriate.
- For each question, mark the correct/most appropriate option/choice by putting a cross ($\times$) mark in the appropriate box of the answer sheet provided to you. In case, a candidate feels that there is multiple correct options/choices, the candidate has to mark the option/choice which he/she feels is the most appropriate/best. In any case, **only one** option/choice has to be marked for each question. More than one option/choice marked in the answer sheet against a question number will be deemed as incorrect.
- If you mark your option/choice at any place other than the box provided, it will not be evaluated.
- Each correct answer carries 2 marks.
- Use of any written/printed material, calculator, docu-pen, any communication devices like cell phones/i-pods etc. inside the examination hall is not allowed. Candidates found with such items will be reported and his/her candidature will be summarily cancelled.

- Blank Sheet(s) for doing rough work/calculations is/are appended at the end of the Test Booklet.

***Warning:** Any malpractice or any attempt to commit any kind of malpractice in the examination hall will disqualify the candidate.*

MULTIPLE-CHOICE QUESTIONS

Throughout this booklet, $\mathbb{N}$ stands for the set of all natural numbers, $\mathbb{Z}$ stands for the set of all integers, $\mathbb{Q}$ stands for the set of all rational numbers, $\mathbb{R}$ stands for the set of all real numbers and $\mathbb{C}$ stands for the set of all complex numbers.

1. How many subsets of size two are there of the set $\{1, 2, \cdots, 20\}$ which do not consist of two consecutive integers ?

   (A) 190 (B) 171 (C) 64 (D) 54

2. Let z, w be non-zero complex numbers satisfying $\overline{z} + i\,\overline{w} = 0$. If $\text{Arg}(zw) = \pi$, then the $\text{Arg}(z) =$ _____.

(A) $\frac{\pi}{4}$. (B) $\frac{\pi}{2}$. (C) $\frac{3\pi}{4}$. (D) $\frac{5\pi}{4}$.

3. The non-zero complex number(s) z satisfying the equations: $|z+1| = 1$ and $|z^2+1| = 1$ is/are _____.

(A) ω and ω^2 (ω is a non-real cube root of unity).

(B) Only ω.

(C) Only ω^2.

(D) None of these.

4. If a non-zero complex number z satisfies $\left|z - \frac{4}{z}\right| = 2$, then the difference between the maximum and the minimum value of $|z|$ is _____.

(A) 1. (B) 2. (C) $\sqrt{5}$. (D) $2\sqrt{5}$.

5. If R and S are two different relations on a set X, then which one of the following is false ?

(A) R and S are transitive, then $R \cup S$ is also transitive.

(B) R and S are reflexive, then $R \cup S$ is also reflexive.

(C) R and S are symmetric, then $R \cup S$ is also symmetric.

(D) R and S are reflexive, then $R \cap S$ is also reflexive.

6. Consider the sets $X = [-1, 1], Y = [0, 1], Z = [-1, 0]$ and define the following relations.

$R_1 = \{(x, y) \in X \times Y : x^2 + y^2 = 1\}$.

$R_2 = \{(x, y) \in X \times Z : x^2 + y^2 = 1\}$.

$R_3 = \{(x, y) \in Y \times Z : x^2 + y^2 = 1\}$.

Pick out the correct option.

(A) Only R_1 is a function. (B) Only R_1 and R_2 are functions.

(C) Only R_2 and R_3 are functions. (D) R_1, R_2 and R_3 are functions

7. If a function f satisfies $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ $(0 \neq x \in \mathbb{R})$, then which one of the following is true for the set $S = \{x \in \mathbb{R} : f(x) = f(-x)\}$?

(A) an empty set. (B) contains exactly one element.

(C) contains exactly two elements. (D) contains more than two elements.

8. If a polynomial $p(x) = ax^2 + bx + c$ (a, b and c are constants with $a \neq 0$) has positive distinct roots which are reciprocals of each other, then which one of the following is correct ?

(A) $p'(1) = 0$. (B) $a\,p'(1) < 0$.

(C) $a\,p'(1) > 0$. (D) Nothing can be said about $a\,p'(1)$.

9. $\lim\limits_{x\to 0} \dfrac{1 - \cos\{1 - cos(1 - cosx)\}}{x^8} =$_____.

(A) $\dfrac{1}{2^2}$. (B) $\dfrac{1}{2^4}$. (C) $\dfrac{1}{2^6}$. (D) $\dfrac{1}{2^8}$.

10. For the function f given by $f(x) = \lim\limits_{n\to\infty} \dfrac{\ln(2+x) - x^{2n}\sin x}{1 + x^{2n}}$ $(x > 0)$, which one of the following is true ?

(A) f is continuous at $x = 1$. (B) $\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$.

(C) $\lim\limits_{x\to 1^-} f(x)$ does not exists. (D) $\lim\limits_{x\to 1^+} f(x) = \sin 1$.

11. For a function $f : \mathbb{R} \longrightarrow \mathbb{R}$, which of the following statement is true (T) or false (F) ?

(i) $|f(x) - f(y)| \leq |x - y|$ $(x, y \in \mathbb{R})$, then f is continuous on $\mathbb{R}$.

(ii) $|f(x) - f(y)| \leq |x - y|$ $(x, y \in \mathbb{R})$, then f is differentiable on $\mathbb{R}$.

(iii) $|f(x) - f(y)| \leq |x - y|^2$ $(x, y \in \mathbb{R})$, then f is continuous and differentiable on $\mathbb{R}$.

(iv) $|f(x) - f(y)| \leq |x - y|^2$ $(x, y \in \mathbb{R})$, then f is a constant function on $\mathbb{R}$.

(A) TTTT (B) TFFT (C) TFTT (D) TFTF

12. If $f(x) = a|\sin x| + be^{|x|} + c|x|^3$ $\left(-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}\right)$, is differentiable at $x = 0$, then which one of the following is true ?

(A) $b + c = 0, a \in \mathbb{R}$. (B) $a + b = 0, c \in \mathbb{R}$. (C) $b = c = 0, a \in \mathbb{R}$. (D) $a = c = 0, b \in \mathbb{R}$.

13. Let $f(x) = \begin{cases} 1 + x, & 0 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 3 \end{cases}$. Then, the point(s) at which the function $(f \circ f)$ is not differentiable is/are _____.

(A) only at $x = 1$. (B) only at $x = 2$.

(C) only at $x = 1$ and $x = 2$. (D) at all the points in $(0, 3)$.

14. If the rate of increase of a side of a triangle inscribed in a circle of radius 3 units is $\sqrt{2}$ times the rate of increase of the measure of the angle opposite to the side of the triangle, then the measure of the angle is _____.

(A) $\dfrac{\pi}{6}$. (B) $\dfrac{\pi}{4}$. (C) $\dfrac{\pi}{3}$. (D) $\dfrac{\pi}{2}$.

15. If $\int_0^{100} f(t)\,dt = 9$, then what is the value of $\sum_{k=1}^{100}\left(\int_0^1 f(k-1+t)\,dt\right)$?

(A) 7. (B) 9. (C) 90. (D) 900.

16. If $\int_0^x f(t)\,dt = x + \int_x^1 tf(t)\,dt + \ln(2) - 1$ $(x > 0)$, then $f(2) =$ _____.

(A) $-\frac{1}{3}$. (B) 0. (C) $\frac{1}{3}$. (D) 1.

17. If $f(x) = \lim_{n\to\infty}\left(x^{\frac{1}{n}} - x^{\frac{1}{n+1}}\right)$ $(x > 0)$, then $\int xf(x)\,dx$=_____.

(A) $\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 + \text{Constant}$. (B) $x^2\log(x) - \frac{1}{4}x^2 + \text{Constant}$.

(C) $\frac{1}{2}x^2\log(x) - x^2 + \text{Constant}$. (D) $\frac{1}{4}x^2\log(x) - \frac{1}{2}x^2 + \text{Constant}$.

18. If $\mathrm{I} = \int \sin^{-1}(x)\,dx$ and $\mathrm{J} = \int \sin^{-1}(\sqrt{1-x^2})\,dx$, then which one of the following is correct ?

(A) $\mathrm{I} = \mathrm{J} + \text{Constant}$. (B) $\mathrm{I} = \frac{\pi}{2}\mathrm{J} + \text{Constant}$.

(C) $\mathrm{I} + \mathrm{J} = \frac{\pi}{2}x + \text{Constant}$. (D) $\mathrm{I} + \mathrm{J} = \frac{\pi}{2} + \text{Constant}$.

19. $\int_1^4 \frac{\sqrt{x}\,dx}{\sqrt{5-x}+\sqrt{x}} =$ _____.

(A) 2. (B) $\frac{3}{2}$. (C) 1. (D) $\frac{1}{2}$.

20. A curve $y = f(x)$ passes through the point $(1, 1)$. If the tangent at $P(x, y)$ cuts the x-axis and the y-axis at the points A and B, respectively satisfying $BP : AP = 3 : 1$, then which one of the following is true ?

(A) The equation of the curve is $xy' - 3y = 0$.

(B) The normal at the point $(1, 1)$ is $x + 3y = 4$.

(C) The curve passes through the point $\left(2, \frac{1}{5}\right)$.

(D) The equation of the curve is $xy' + 3y = 0$.

21. Consider the following statements and pick out the correct option.

I. The principal branch of $\cos^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

II. The principal branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

III. The principal branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(A) Only III is false. (B) Both I and II are false.

(C) Both II and III are true. (D) Both I and III are true.

22. What is the measure of the angle between the curves: $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 5$? (Here $[\cdot]$ stands for the greatest integer function).

(A) $2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ (C) $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (D) $2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

23. Which one of the functions given below satisfy all the conditions of the Rolle's theorem ?(Here $[\cdot]$ stands for the greatest integer function).

(A) $f(x) = 1 - |x - 1|, -1 \leq x \leq 3.$ (B) $f(x) = [x] + [-x], -1 \leq x \leq 1.$

(C) $f(x) = |x|\,[x], -1 \leq x \leq 1.$ (D) $f(x) = |x| - |\sin x|, -1 \leq x \leq 1.$

24. If the function $f(x) = (x-3)^2$ satisfies all the conditions of the Mean Value Theorem, then a point on the graph of the function f, where the tangent is parallel to the chord joining the points $(3, 0)$ and $(4, 1)$ is _____.

(A) $(1, 4)$. (B) $(4, 1)$. (C) $\left(\frac{7}{2}, \frac{1}{4}\right)$. (D) $\left(\frac{7}{2}, \frac{1}{2}\right)$.

25. Let $f : [-1, 2] \longrightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. If $R_1 = \int_{-1}^{2} xf(x)\,dx$ and R_2 is the area of the region bounded by $y = f(x), x = -1, x = 2$ and the x-axis, then which one of the following is true?

(A) $2R_1 = R_2$. (B) $R_1 = 2R_2$. (C) $3R_1 = 2R_2$. (D) $2R_1 = 3R_2$.

26. A square is formed by the lines $x = 4, y = 4$ and the co-ordinate axes. If the parabolas $y^2 = 4x, x^2 = 4y$ divide the square region into three parts with areas A_1, A_2 and A_3, respectively (numbered from top to bottom), then $A_1 : A_2 : A_3$ is _____.

(A) 1 : 2 : 1 (B) 2 : 1 : 2 (C) 1 : 1 : 1 (D) 1 : 2 : 3

27. The differential equation: $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) fixed radius 1 and variable centres along the x-axis.

(B) fixed radius 1 and variable centres along the y-axis.

(C) variable radii and a fixed centre at $(0, 1)$.

(D) variable radii and a fixed centre at $(0, -1)$.

28. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation: $y(1 + xy)dx = x\,dy$, then $f\left(-\frac{1}{2}\right) =$_____.

(A) $-\frac{2}{5}$. (B) $-\frac{4}{5}$. (C) $\frac{2}{5}$. (D) $\frac{4}{5}$.

29. If the vectors $\vec{a}, \vec{b}, \vec{c}$ represent the sides BC, CA and AB of a triangle $\triangle ABC$, then which one of the following is true ?

(A) $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$.
(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.
(C) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$.
(D) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$.

30. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are unit vectors satisfying $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$ and $\vec{a} \cdot \vec{b} = \frac{1}{2}$, then which one of the following is true ?

(A) $\vec{a}, \vec{b}, \vec{c}$ are not coplanar.
(B) $\vec{b}, \vec{c}, \vec{d}$ are not coplanar.
(C) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel.
(D) $\vec{b}$ and $\vec{d}$ are not parallel.

31. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 7, |\vec{b}| = 5$ and $|\vec{c}| = 3$, then the angle between the vectors $\vec{b}$ and $\vec{c}$ is _____.

(A) $\frac{\pi}{6}$.
(B) $\frac{\pi}{4}$.
(C) $\frac{\pi}{3}$.
(D) $\frac{\pi}{2}$.

32. The equation of the line parallel to the x-axis and intersecting the curve $y = \sqrt{x}$ at an angle with measure $\frac{\pi}{4}$ is _____.

(A) $y = \frac{1}{8}$.
(B) $y = \frac{1}{4}$.
(C) $y = \frac{1}{2}$.
(D) $y = 1$.

33. Which one of the following is the image of the point $(3, 0, 2)$ with respect to the line $\frac{x+4}{3} = \frac{y+3}{5} = \frac{z-1}{2}$?

(A) $(-1, 2, 3)$.
(B) $(-5, 4, 4)$.
(C) $(-3, 0, -2)$.
(D) $(-4, 2, 1)$.

34. The equation of the plane passing through the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z - 12 = 0$ is _____.

(A) $2x - 9y + 5z + 4 = 0$.
(B) $3x - 6y + 5z + 4 = 0$.
(C) $5x - 2y + 9z + 4 = 0$.
(D) $9x - 2y - 5z + 4 = 0$.

35. From a group of 7 men and 6 women, 5 persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done ?

(A) 756.
(B) 726.
(C) 720.
(D) 702.

36. The set $X = \{1, 2, 3, \cdots, 12\}$ is to be partitioned into three sets S_1, S_2 and S_3 of equal size such that $S_1 \cap S_2 = S_2 \cap S_3 = S_1 \cap S_3 = \emptyset$. Then, the number of ways to partition the set X is _____.

(A) $\frac{(12)!}{(4!)^3}$.
(B) $\frac{(12)!}{(3!)^3}$.
(C) $\frac{(12)!}{(3!)^4}$.
(D) $\frac{(12)!}{3!(4!)^3}$.

37. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. If p denotes the probability that 7 comes before 5, then what is the value of p ?

(A) $\frac{4}{5}$. (B) $\frac{3}{5}$. (C) $\frac{2}{5}$. (D) $\frac{1}{5}$.

38. Consider the following statements, where A, B and C are three mutually independent events.

I. A and $B \cup C$ are independent.

II. A and $B \cap C$ are independent.

Then, pick out the correct option.

(A) Only I is true. (B) Only II is true.

(C) Both I and II are true. (D) Neither I nor II is true.

39. Which one of the following is the equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$?

(A) $y = x + 2$. (B) $y = 3x + \frac{2}{3}$. (C) $y = 2x + 3$. (D) $y = \frac{1}{2}x + 2$.

40. If a circle with center on the focus of the parabola $y^2 = 4x$ touches the directrix of the parabola, then a point of intersection of the circle and the parabola is _____.

(A) $(-1, \pm 2)$. (B) $(1, \pm 2)$. (C) $(-2, \pm 1)$. (D) $(2, \pm 1)$.

41. Let P, Q denote the foci of an ellipse and R denote one of the end point of the minor axis. If the $\triangle PQR$ is equilateral, then what is the eccentricity of the ellipse ?

(A) $\frac{1}{4}$. (B) $\frac{1}{3}$. (C) $\frac{1}{2}$. (D) $\frac{3}{4}$.

42. If e_1 and e_2 are eccentricities of a hyperbola and its conjugate respectively, then which one of the following is true ?

(A) $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$. (B) $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 2$. (C) $\frac{1}{e_1} + \frac{1}{e_2} = 1$. (D) $\frac{1}{e_1} + \frac{1}{e_2} = 2$.

43. A tangent having slope $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$ meets the major and the minor axes at the points P and Q, respectively. If O is the centre of the ellipse, then what is the area of the $\triangle POQ$?

(A) 16 sq. units. (B) 20 sq. units. (C) 24 sq. units. (D) 28 sq. units.

44. If $a_1, a_2, a_3, \cdots$ are terms of an arithmetic progression (A.P.), $S_1 = a_2 + a_4 + a_6 + \cdots$ up to 100 terms and $S_2 = a_1 + a_3 + a_5 + \cdots$ up to 100 terms, then the common difference of the A.P. is _____.

(A) $\frac{|S_1 - S_2|}{100}$. (B) $\frac{|S_1 - S_2|}{50}$. (C) $\frac{|S_1 - S_2|}{2}$. (D) $|S_1 - S_2|$.

45. If $a_1, a_2, a_3, \cdots, a_{20}$ are terms of an A.P. and $a_1 + a_{20} = 45$, then $a_1 + a_2 + a_3 + \cdots + a_{20}$=_____.

(A) 450. (B) 350. (C) 250. (D) 90.

46. The sum of the series: $\frac{1}{1!} + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \cdots$=_____.

(A) $e - 1$. (B) $e^2 - 1$. (C) $2e^2 - 1$. (D) $e(e-1)$.

47. For a sequence $\{a_n\}_{n\geq 1}$, let $a_1 = 2$ and $3a_{n+1} - a_n = 0$, then what is the sum of the series $\sum_{n=1}^{\infty} a_n$?

(A) $\frac{13}{3}$. (B) $\frac{9}{2}$.

(C) 3. (D) can not be determined.

48. The coefficient of x^5 in the binomial expansion of $(1+x)^{21}+(1+x)^{22}+\cdots+(1+x)^{30}$ is _____.

(A) $3\binom{31}{6} + 2\binom{21}{6}$ (B) $2\binom{31}{6} + 3\binom{21}{6}$ (C) $\binom{31}{6} + \binom{21}{6}$ (D) $\binom{31}{6} - \binom{21}{6}$

49. Sum of the coefficients of the last 15 terms in the expansion of $(1+x)^{29}$ is _____.

(A) 2^{27}. (B) 2^{28}. (C) 2^{29}. (D) 2^{30}.

50. Let α and β be the roots of the equation $x^2 + px + p^3 = 0$ $(p \neq 0)$ such that (α, β) lies on the parabola $y = x^2$, then (α, β)=_____.

(A) $(4, -2)$. (B) $(-4, 2)$. (C) $(-4, -2)$. (D) $(4, 2)$.