INSTITUTE OF MATHEMATICS AND APPLICATIONS, BHUBANESWAR
ENTRANCE TEST FOR ADMISSION INTO B.Sc.(HONOURS) IN MATHEMATICS & COMPUTING, 2021-2022

Maximum Marks: 100 **Time: 2 hours**

Please read, carefully, the instructions that follow.

• This question booklet contains 08 printed pages with 50 multiple choice questions.
• Check that the booklet does not have any un-printed or torn or missing pages or items etc., if so, get it replaced by a complete test booklet before attempting to answer. No extra time will be given, if replaced afterwards.
• Each of the questions/incomplete statements is followed by ***four*** options/choices marked as **(A)**, **(B)**, **(C)**, **(D)** under each question/statement, of which only ***one*** of them is correct/most appropriate.
• For each question, mark the correct/most appropriate option/choice by putting a cross ($\times$) mark in the appropriate box of the answer sheet provided to you. If you write your choice at any place other than the box provided, it will not be evaluated.
• More than one choice marked against a question number will be deemed as incorrect.
• Each correct answer carries 2 marks.
• Use of calculator, log table, mobile phone or any electronic gadget in the examination is not allowed.

Warning: Any malpractice or any attempt to commit any kind of disturbances in the examination hall/during the examination will disqualify the candidature.

MULTIPLE-CHOICE QUESTIONS

Throughout this booklet, $\mathbb{R}$ stands for the set of all real numbers and $\mathbb{C}$ stands for the set of all complex numbers.

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1. Let A be a non-empty set and $\mathcal{R}_1, \mathcal{R}_2 \subseteq A \times A$. Consider the following statements.

   S_1. If $\mathcal{R}_1, \mathcal{R}_2$ are reflexive relations, then $\mathcal{R}_1 \cap \mathcal{R}_2$ and $\mathcal{R}_1 \cup \mathcal{R}_2$ are reflexive relations.

   S_2. If $\mathcal{R}_1, \mathcal{R}_2$ are symmetric relations, then $\mathcal{R}_1 \cap \mathcal{R}_2$ and $\mathcal{R}_1 \cup \mathcal{R}_2$ are symmetric relations.

   S_3. If $\mathcal{R}_1, \mathcal{R}_2$ are transitive relations, then $\mathcal{R}_1 \cap \mathcal{R}_2$ and $\mathcal{R}_1 \cup \mathcal{R}_2$ are transitive relations.

   Which of the above statement(s) is/are false ?

   (A) S_2 only. (B) S_3 only. (C) S_2 and S_3 only. (D) S_1, S_2 and S_3.

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2. Let $f(x) = \min\{\sin x, \cos x\}, x \in \mathbb{R}$. Then, the range of $g(x) = [f(x)]$ ($[x]$ stands for the greatest integer $\leq x$) is equal to:

   (A) $\{0\}$. (B) $\{-1, 0\}$. (C) $\{0, 1\}$. (D) $\{-1, 0, 1\}$.

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3. In a certain town, 45% families own a mobile phone, 15% own a car and 60% families own neither a mobile phone nor a car. 2000 families own both a mobile phone and a car.

   Consider the following statements.

   I. 15% families own both a mobile phone and a car.
   II. 40% families own either a mobile phone or a car.
   III. 25,000 families live in the town.

   Which of the above statement(s) is/are correct ?

   (A) II only.
   (B) II and III only.
   (C) I and III only.
   (D) All the above statements are correct.

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4. For the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$, which one of the following options is true ?

   (A) f is an one-one, but not an onto function.
   (B) f is not an one-one, but an onto function.
   (C) f is both an one-one and an onto function.
   (D) f is neither an one-one nor an onto function.

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5. Let z be a complex number satisfying the equation $|z - (1+i)| = \sqrt{2}$. Then the locus traced by the complex number $\dfrac{2}{z}$ in the complex plane $\mathbb{C}$ is

   (A) $x + y + 1 = 0$.
   (B) $x - y - 1 = 0$.
   (C) $x - y + 1 = 0$.
   (D) $x + y - 1 = 0$.

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6. Let $E_1 = \{z \in \mathbb{C} : |z+2| + |z-2| = 8\}$ and $E_2 = \{z \in \mathbb{C} : |z+2i| + |z-2i| = 8\}$. Then $E_1 \cup E_2$

   (A) is an empty set.
   (B) is a singleton set.
   (C) has exactly 4 elements.
   (D) has infinitely many elements.

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7. The value of the expression $2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+4\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)+\cdots+(n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$, where ω is the imaginary cube root of unity, is equal to:

   (A) $\dfrac{n(n^2+1)}{4}$.
   (B) $\dfrac{n(n^2+2)}{4}$.
   (C) $n + \dfrac{n^2(n+1)^2}{4}$.
   (D) None of these.

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8. If $S_n = \dfrac{1}{1^3} + \dfrac{1+2}{1^3+2^3} + \dfrac{1+2+3}{1^3+2^3+3^3} + \ldots + \dfrac{1+2+\ldots+n}{1^3+2^3+\ldots+n^3}$, then $\lim\limits_{n\to\infty}$ equals to:

   (A) 2.
   (B) 3.
   (C) $\dfrac{1}{2}$.
   (D) 1.

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9. Let $x_1, x_2, x_3, \ldots, x_n, \ldots$ be the terms of an A.P. If $x_3 + x_7 + x_{11} + x_{15} = 40$, then $x_1 + x_2 + x_3 + \ldots + x_{17}$ is equal to:

(A) 340. (B) 306. (C) 204. (D) 170.

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10. Let a, b, c be positive real numbers such that a^x, b^x and c^x are in G.P. Consider the following statements.

I. a, b, c are in A.P.
II. $\log a, \log b, \log c$ are in A.P.
III. a, b, c are in G.P.
IV. $\log a, \log b, \log c$ are in G.P.

Which of the above statement(s) is/are correct ?

(A) III only. (B) II and IV only. (C) II and III only. (D) I and IV only.

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11. If $C_0, C_1, C_2, \ldots, C_n$($n$ is a multiple of 2) are binomial coefficients of $(1+x)^n$, then the value of $2C_1 + 2^3C_3 + 2^5C_5 + \cdots + 2^nC_n$ is equal to:

(A) $\dfrac{3^n + (-1)^n}{2}$. (B) $\dfrac{3^n - (-1)^n}{2}$. (C) $\dfrac{3^n + 1}{2}$. (D) $\dfrac{3^n - 1}{2}$.

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12. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - bx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then, the common root of these equations is equal to:

(A) 4. (B) 3. (C) 2. (D) 1.

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13. Let α and β be the roots of the equation $ax^2 + bx + c = 0$. If $\beta < \alpha < 0$, then the quadratic equation whose roots are $|\alpha|$ and $|\beta|$ is equal to:

(A) $|a|x^2 + |b|x + |c| = 0$. (B) $ax^2 - |b|x + c = 0$.

(C) $|a|x^2 - |b|x + |c| = 0$. (D) $|a|x^2 + |b|x - |c| = 0$.

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14. $\lim\limits_{x\to 0} \dfrac{\sin(\pi \cos^2 x)}{x^2}$ is equal to:

(A) π. (B) $\dfrac{\pi}{2}$. (C) 1. (D) -π.

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15. If $\alpha = \lim\limits_{x\to 0} \dfrac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $\beta = \lim\limits_{x\to 0} \dfrac{[\,|x|\,]}{x}$($[\cdot]$ is the greatest integer function), then

(A) $\alpha = 0$ and $\beta = 1$. (B) α does not exists and $\beta = 0$.

(C) $\alpha = 1$ and $\beta = 0$. (D) α does not exists and $\beta = 1$.

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16. Pick out the correct option for the function f defined on $\mathbb{R}$ by $f(x) = \min\{|x|, |x-2|, 2-|x-1|\}$.

(A) f is continuous, differential for all $x \in \mathbb{R}$.

(B) f is continuous for all $x \in \mathbb{R}$, but not differential at 3 points of $\mathbb{R}$.

(C) f is continuous for all $x \in \mathbb{R}$, but not differential at 5 points of $\mathbb{R}$.

(D) The maximum value of f is 2.

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17. If the function $f(x) = a|\sin x| + be^{|x|} + c|x|^3$ is differentiable at $x = 0$, then

(A) $a \neq 0, b \neq 0, c \in \mathbb{R}$.

(B) $a = c = 0, b \in \mathbb{R}$.

(C) $b = c = 0, a \in \mathbb{R}$.

(D) $a = b = 0, c \in \mathbb{R}$.

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18. Consider the following statements for the function $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \leq x \leq 2 \\ 37 - x, & 2 < x \leq 3 \end{cases}$.

S_1. f is increasing on $[-1, 2]$.
S_2. f is continuous on $[-1, 3]$.
S_3. f' exists at $x = 2$.
S_4. f has the maximum value at $x = 2$.

Which of the above statement(s) is/are correct ?

(A) S_1 and S_2 only.

(B) S_3 and S_4 only.

(C) S_1, S_2 and S_4 only.

(D) S_1, S_3 and S_4. only.

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19. The point on the curve $y = x^3 - 3x^2 + 2x$ at which the tangent to the curve is parallel to the line $y = 2x - 3$ is:

(A) $(0, 2)$.

(B) $(2, 0)$.

(C) $(2, 2)$.

(D) $(2, -1)$.

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20. Pick out the correct option for the function $f(x) = x + \sin x, x \in \mathbb{R}$.

(A) f has a minimum, but no maximum .

(B) f has a maximum, but no minimum.

(C) f has neither a minimum nor a maximum.

(D) f has both a minimum and a maximum.

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21. If $A + B + C = 180^\circ$ and $\cos A + \cos B + \cos C = \sin A + \sin B + \sin C = 0$, then the value of $\cos(3A) + \cos(3B) + \cos(3C)$ is equal to:

(A) 3.

(B) $\sqrt{3}$.

(C) 0.

(D) -3.

22. The smallest positive value of θ which satisfies the equation $2\sin^2(\theta) + \sqrt{3}\cos(\theta) + 1 = 0$ is

(A) $\frac{\pi}{6}$.

(B) $\frac{\pi}{3}$.

(C) $\frac{2\pi}{3}$.

(D) $\frac{5\pi}{6}$.

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23. Consider the following statements.

S_1. $\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right) = 0.$

S_2. $\cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right) = 0.$

S_3. $\cos^3\left(\frac{\pi}{8}\right) + \cos^3\left(\frac{3\pi}{8}\right) + \cos^3\left(\frac{5\pi}{8}\right) + \cos^3\left(\frac{7\pi}{8}\right) = 0.$

Which of the above statement(s) is/are correct ?

(A) S_1 only. (B) S_2 and S_3 only. (C) S_1 and S_3 only. (D) S_1, S_2 and S_3.

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24. If $\cos(2\tan^{-1} x) = \frac{1}{2}$, then the value of x is equal to:

(A) $\frac{1}{\sqrt{3}}$. (B) $\sqrt{3}$. (C) $1 - \frac{1}{\sqrt{3}}$. (D) $1 - \sqrt{3}$.

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25. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $xy + yz + xz$ is equal to:

(A) -3. (B) 3. (C) 0. (D) 1.

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26. From 6 men and 4 ladies, a committee of 5 members is to be formed. In how many ways this can be done, if the committee is to include at least one lady ?

(A) 340. (B) 315. (C) 290. (D) 246.

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27. Let A and B be two events such that $\text{Prob}(\overline{A} \cup B) = \frac{1}{6}$, $\text{Prob}(A \cap B) = \frac{1}{4}$ and $\text{Prob}(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then, the events A and B are

(A) mutually exclusive and independent. (B) equally likely, but not independent.

(C) independent, but not equally likely. (D) independent and equally likely.

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28. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If k_1, k_2 and k_3 are the numbers obtained on the die, then probability that $\omega^{k_1} + \omega^{k_2} + \omega^{k_3} = 0$ is equal to:

(A) $\frac{1}{18}$. (B) $\frac{1}{9}$. (C) $\frac{2}{9}$. (D) $\frac{1}{36}$.

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29. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits $1, 2$ and 3 only is equal to:

(A) 84. (B) 77. (C) 42. (D) 35.

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30. If $\int \frac{\tan x}{1+tanx+\tan^2 x}dx = x - \frac{\alpha}{\sqrt{\beta}}\tan^{-1}\left(\frac{1+\alpha\tan x}{\sqrt{\beta}}\right)+C$, then the ordered pair (α, β) equals to:

(A) $(2, 1)$. (B) $(-2, 3)$. (C) $(2, 3)$. (D) $(-2, 1)$.

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31. Let f be a continuous function on $\mathbb{R}$. If $I_1 = \int_0^{3\pi} f(\cos^2 x)\,dx$, $I_2 = \int_0^{2\pi} f(\cos^2 x)\,dx$ and $I_3 = \int_0^{\pi} f(\cos^2 x)\,dx$, then which one of the following options is correct ?

(A) $I_1 - I_2 - I_3 = 0$. (B) $I_1 + 2I_2 + 3I_3 = 0$. (C) $I_1 - 2I_2 - I_3 = 0$. (D) $I_1 - 2I_3 = 0$.

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32. Consider the following statements.

S_1. The area enclosed by the ellipse $E : \frac{x^2}{3} + \frac{y^2}{2} = 1$ is less than the area bounded by the curve $R : \frac{|x|}{3} + \frac{|y|}{2} = 1$.

S_2. The length of the major axis of the ellipse E is more than the distance between the points of R on the x-axis.

Which of the above statements is/are correct ?

(A) Both S_1 and S_2 are true. (B) S_1 is true, but S_2 is false.

(C) S_1 is false, but S_2 is true. (D) Both S_1 and S_2 are false.

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33. The ratio of the areas of the incircle and circumcircle of an equilateral triangle is equal to:

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 1 : 4

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34. If $\overrightarrow{a}, \overrightarrow{b}$ are two vectors such that $\overrightarrow{a} \cdot \overrightarrow{b = 0}$ and $\overrightarrow{a} \times \overrightarrow{b = 0}$, then which one of the following options is correct ?

(A) $\overrightarrow{a}$ is perpendicular to $\overrightarrow{b}$. (B) $\overrightarrow{a}$ is parallel to $\overrightarrow{b}$.

(C) Either $\overrightarrow{a}$ or $\overrightarrow{b}$ is a null vector. (D) Both $\overrightarrow{a}$ and $\overrightarrow{b}$ are non-zero vector.

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35. If $\overrightarrow{a} = \hat{j} - \hat{k}$, $\overrightarrow{c} = \hat{i} - \hat{j} - \hat{k}$, then the vector $\overrightarrow{b}$ satisfying $\overrightarrow{a} \times \overrightarrow{b} + \overrightarrow{c = 0}$ and $\overrightarrow{a} \cdot \overrightarrow{b} = 3$ is equal to:

(A) $-\hat{i} + \hat{j} - 2\hat{k}$. (B) $2\hat{i} - \hat{j} + 2\hat{k}$. (C) $\hat{i} - \hat{j} - 2\hat{k}$. (D) $\hat{i} + \hat{j} - 2\hat{k}$.

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36. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following relation holds for $n \geq 1$?

(A) $A^n = 2^{n-1}A - (n-1)I$.

(B) $A^n = nA - (n-1)I$.

(C) $A^n = 2^{n-1}A + (n-1)I$.

(D) $A^n = nA + (n-1)I$.

37. Let $A = [a_{ij}]$, $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = 3^{(i+j-2)}a_{ij}$ $(1 \leq i, j \leq 3)$ and $\det(B) = 81$. Then, the value of the determinant of the matrix A is equal to:

(A) 3.

(B) $\frac{1}{3^4}$.

(C) $\frac{1}{3^3}$.

(D) $\frac{1}{3^2}$.

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38. The system of linear equations $x + \lambda y - z = 0$, $\lambda x - y - z = 0$ and $x + y - \lambda z = 0$ has a non-trivial solution for

(A) exactly one value of λ.

(B) exactly two values of λ.

(C) exactly three values of λ.

(D) infinitely many values of λ.

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39. A circle passes through the point $(-2, 4)$ and touches the y-axis at $(0, 2)$. Which one of following equations represents a diameter of the circle ?

(A) $2x - 3y + 10 = 0$. (B) $4x + 5y - 6 = 0$. (C) $5x + 2y + 4 = 0$. (D) $3x + 4y - 3 = 0$.

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40. Let C_1 be a circle with center $(1, 1)$ and radius 1 unit. Let C_2 be a circle with center $(0, \alpha)$, passing through the origin and touching the circle C_1 externally. Then, the radius of the circle C_2 is equal to:

(A) $\frac{\sqrt{3}}{2}$units.

(B) $\sqrt{\frac{3}{2}}$units.

(C) $\frac{1}{2}$unit.

(D) $\frac{1}{4}$unit.

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41. If the four points of intersection of the straight lines $2x - y + 1 = 0$ and $x - 2y + 3 = 0$ with the co-ordinate axes are concyclic, then the center of the circle is equal to:

(A) $\left(\frac{7}{4}, \frac{5}{4}\right)$.

(B) $\left(\frac{7}{4}, -\frac{5}{4}\right)$.

(C) $\left(-\frac{7}{4}, \frac{5}{4}\right)$.

(D) $\left(-\frac{7}{4}, -\frac{5}{4}\right)$.

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42. If $ABCDEF$ is a regular hexagon and $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AE} + \overrightarrow{AF} = \lambda \overrightarrow{AD}$, then the value of λ is equal to:

(A) 5.

(B) 4.

(C) 3.

(D) 2.

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43. Which one of the following is the image of the point $(1, 3, 4)$ with respect to the plane $2x - y + z = -3$?

(A) $(3, 5, -2)$.

(B) $(-3, 5, 2)$.

(C) $(3, -5, 2)$.

(D) $(3, 5, 2)$.

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44. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $(y-2)^2 = 4(x+1)$. After reflection , the ray must pass through which one of the following points ?

(A) $(-2, 0)$. (B) $(-1, 2)$. (C) $(0, 2)$. (D) $(2, 0)$.

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45. An ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the co-ordinate axes. Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R. Then, the eccentricity of the ellipse E_2 is equal to:

(A) $\frac{\sqrt{3}}{2}$. (B) $\frac{1}{2}$. (C) $\frac{\sqrt{2}}{2}$. (D) $\frac{1}{4}$.

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46. The locus of a point $P(\alpha, \beta)$ in the plane moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(A) an ellipse. (B) a parabola. (C) a circle. (D) a hyperbola.

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47. The equation of the plane through the line of intersection of the planes $x+2y-3 = 0, y-2z+1 = 0$ and perpendicular to the plane $x+2y-3 = 0$ is:

(A) $2x - y + 7z - 11 = 0$. (B) $2x - y + 10z - 11 = 0$.

(C) $2x - y - 9z - 10 = 0$. (D) $2x - y - 10z - 9 = 0$.

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48. The differential equations of all parabolas having the directrix parallel to the x-axis is

(A) $\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^2 = 0$. (B) $\frac{d^3x}{dy^3} = 0$.

(C) $\frac{d^3y}{dx^3} = 0$. (D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

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49. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then the value of $y(\log 2)$ is equal to:

(A) 7. (B) -3. (C) 5. (D) -2.

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50. Shoes manufacturing company only manufactures shoes for adults. Company wants to know the most popular size which are in demand. Which type of central tendency will be most appropriate for it?

(A) Median (B) Mean (C) Mode (D) Range
